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BEHAVIOR OF THE SECOND ORDER PHASE AUTOMATIC CONTROL SYSTEM,(U)

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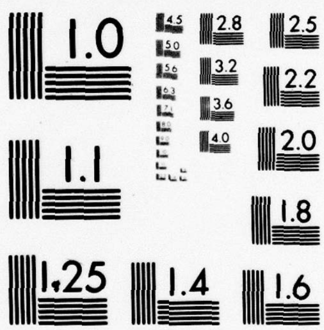

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FOREIGN TECHNOLOGY DIVISION

BEHAVIOR OF THE SECOND ORDER PHASE AUTOMATIC CONTROL SYSTEM

By

H. D. Bettac, and K. H. Schmelovsky

EDITED TRANSLATION

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BEHAVIOR OF THE SECOND ORDER PHASE AUTOMATIC CONTROL SYSTEM

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Part 2

Report from the Central Institute for Solar and Terrestrial Physics of the German Academy of Sciences

Submitted 28 December 1970

5. Acquisition with a Noisy Input Signal

For subsequent observations it is assumed that the frequency-constant input signal is superimposed by a stationary, white, Gaussian narrow-band noise process n(t), whose simple mean

value $\overline{n(t)}$ vanishes and whose unilateral spectral output density N_0 is known. If one additionally assumes that the operating mode of the multiplier is independent of the signal-to-noise ratio applied to it then, because of the division of the Ranlauf process into two partial processes (described in §3), the noise can make itself apparent in this process in only two ways. First by assuming that the error voltage is not only very generally a function of the noise, but a continuous component contained in it specially. Following its calculation Equation (16) could then be solved numerically, or after its rearrangement $(\Delta\omega_0)$ determined from it. Second, there is the possibility that the continuous component is independent of the noise and therewith both are present next to each other at the output of the multiplier. In addition one considers the noise term as being connected into the circuit as shown in Fig. 7 only after completed continuous component formation. Which of the two partial processes is primarily affected by the noise cannot be determined ahead of time so that both possibilities must be calculated. Using the method of calculation of §4 we shall look at the first possibility. It is assumed that the noise noticeably affects the size of the continuous component, so that a noise-dependent continuous component is to be determined below.

5.1. The Noise-Dependent Continuous Component

The equation for the automatic control system for the circuit of the first order thus reads

$$\frac{d\Phi}{dt} = \Delta\omega_0 - a \cdot K \left\{ \sin\Phi(t) + \frac{1}{A} n'(t) \right\} \tag{23}$$

where n'(t) is a noise process resulting from n(t) in a unique manner and whose static properties are known to the extent that one knows them from n(t). Since $\Phi(t)$ represents a random variable, the continuous component formation can no longer be undertaken through the temporal mean value, but must result, on the basis of the stationary nature of n'(t) and therewith of the ergodic theorem, through the ensemble mean value

$$< \sin\Phi > = \int_{-\infty}^{+\infty} \sin\Phi \cdot P(\Phi) d\Phi \tag{24}$$

For its calculation we first of all need the probability function P(, t) of the phase error. Since the change of $P(\Phi, t)$ is dependent only on the instantaneous values of $\Phi(t)$ and n'(t), because n'(t) is a stationary and essentially white Gaussian process, the process described by Eq. (23) describes a Markov process. We can even speak of a continuous Markov-process because the attainable course of (t) in the noise-free case as well as the arrangement of n'(t) in Eq.

(23) permit the conclusion that the probability of the state of the system changing noticeably in a very short time interval is very small.

A complete probability-theoretical description of such a memoryless process is obtained by the determination of the probability density function $P(\Phi, t/\Phi_0, t_0)$ which is constantly conditional for it. From the knowledge of the state Φ_0 of the process at some point in time t_0 the probability is to be found that at a later point in time t it will be in state Φ. The selection of point t_0 is random. One needs only to know how far t is from t_0. For the sake of simplicity the state $\Phi(t_0) = \Phi_0$ can be determined from the initial conditions. It was shown in [3] that the probability density function of a continuous Markov process, by making certain assumptions, can be determined from a fundamental partial differential equation. Here we shall not ask when this is the case, but rather will attempt to find its solution, whereby the assumptions are considered as secondary conditions for the solution. Thus in the following from

(25) $$\frac{\partial P(\Phi,t)}{\partial t} = \sum_{1}^{\infty} n \frac{(-1)^n}{n!} \frac{\partial^n}{\partial \Phi^n} [A_n(\Phi) \cdot P(\Phi,t)]$$

with $P(\Phi,t) \geq 0$ for all Φ and t and

$\int_{-\infty}^{+\infty} P(\Phi, t)\, d\Phi = 1$ for all t

the density P(Φ, t) is to be determined. The A_n represent the boundary values of the n-th moments of the growth process $\Delta\Phi$ in the time Δt for $\Delta t \to 0$. According to [4], for them we write

(26)
$$A_n(\Phi) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{-\infty}^{+\infty} (\Delta\Phi)^n \cdot P(\Delta\Phi, \Delta t) \cdot d(\Delta\Phi); n \geq 1$$
$$= \lim_{\Delta t \to 0} \frac{1}{\Delta t} M\{(\Delta\Phi)^n\} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \overline{(\Delta\Phi)^n}$$

whereby M{} is the mathematical expectation of the particular expression which is in curly brackets. Since the $A_n(\Phi)$ vanish for n > 2 for processes like those given by Eq. (23) (which can easily be checked) Eq. (25) simplifies to

(27)
$$\frac{\partial P(\Phi, t)}{\partial t} = -\frac{\partial}{\partial \Phi}\{A_1(\Phi) \cdot P(\Phi, t)\} + \frac{1}{2} \frac{\partial^2}{\partial \Phi^2}\{A_2(\Phi) P(\Phi, t)\}$$

This equation structure for density distribution is known in the literature as the Fokker-Planck equation. For its solution $A_1(\Phi)$ and $A_2(\Phi)$ must first be determined. From Eq. (23) according to the rule of Eq. (26) these values are calculated as

(28)
$$A_1 = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \overline{\Delta\Phi} = \Delta\omega_0 - a \cdot K \sin\Phi$$

and

(29)
$$A_2 = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \overline{\Delta \Phi^2} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \frac{a^2 \cdot K^2}{A_0} \int_{t-\Delta t}^{t} \int_{t-\Delta t}^{t} M \{n'(u) \cdot n'(v)\} \, du \, dv = \frac{a^2 \cdot K \cdot N_0}{2 A^2}$$

In addition to the standardizing condition the boundary conditions must also be determined for $P(\Phi, t)$. Since for the instantaneous state of the system it makes no difference how often it is inverted at this point in time, it is sufficient to solve equation (27) in the range $-\pi \leq \Phi \leq +\pi$, i.e., $P(\; , t)$ is periodic in 2π. It can then be shown that if $P(\Phi, t)$ is a solution of Eq. (27), then it is also satisfied by $P(\Phi + 2n\pi; t)$. Since with the determination of $P(\Phi, t)$ from Eq. (27) one makes a first-order circuit, it is sufficient to know only the stationary solution, because not only for $\Delta\omega_0 = 0$ but also for $\Delta\omega_0 \neq 0$ the distribution density function is independent of time. If $\Delta\omega_0 < \Delta\omega_L$ then Φ needs to change at most by the value 2π in order to reach a stable point Φ_{ST} in the sense of

(30)
$$\Phi_{ST} = \arcsin \frac{\Delta\omega}{a K}$$

With a stationary noise process in the middle this process always

occurs in the same manner. If $\Delta\omega_0 > \Delta\omega_L$ then ϕ can certainly run over many 2π; since it doesn't result in acquisition, however, all periods are of equal value, i.e. even in this case the distribution of all ϕ in $-\pi \leq \phi \leq +\pi$ is independent of time. The difference between the individual distribution functions will lie only in their symmetry to $\Phi = 0$ because for $\Delta\omega_0 \neq 0$ it must be expected that the probability of stopping for all possible Φ in the vicinity of the particular stable points is the greatest [5]. Indeed for $\Delta\omega_0 > \Delta\omega_L$ there are no more stable points; nevertheless the obtained distribution densities are unique. For an increasing $\Delta\omega_0$ they increasingly approach a sinusoidal curve.

With

$$\frac{\partial P(\phi,t)}{\partial t} = 0$$

and division by the factor in front of $\partial^2 P/\partial\phi^2$ the Fokker-Planck equation which is to be solved is simplified to

(31)
$$0 = \frac{d}{d\phi}[(\varepsilon \sin\phi - \beta)\, P(\phi)] + \frac{d^2 P(\phi)}{d\phi^2}$$

with

(32)
$$\varepsilon = \frac{4A^2}{a \cdot K \cdot N_0}$$

and

(33) $$\beta = \varepsilon \cdot \frac{\Delta\omega_0}{\Delta\omega_L} = \varepsilon \cdot \frac{1}{\gamma}$$

The solution of Eq. (31) can take place in closed form (then the numerically calculated distribution densities as a function of ε and β also show the behavior described above), however, the selection of the step width turns out to be quite critical with the large amounts of the arguments of the exponential functions inserted in $P(\Phi)$. Since they must be chosen very small so that high computer costs would arise [sic], an attempt was made to construct an approximate solution of somewhat simpler structure for $P(\Phi)$. With $\gamma = \varepsilon/\beta$ for most of the observed cases a smaller parameter is present so that it is possible to solve Eq. (31) with the aid of a calculation of a perturbation whereby γ can serve as a perturbation parameter. From a one-time integration and transformation for Eq. (31) one obtains

(34) $$C_1 + P(\Phi) = \gamma \left\{ \sin\Phi \cdot P(\Phi) + \frac{1}{\varepsilon} \frac{dP(\Phi)}{d\Phi} \right\}$$

If one proceeds with the equation

$$P(\Phi, \varepsilon, \gamma) = \sum_{0}^{\infty} i \gamma^i P_i(\Phi, \varepsilon)$$

in Eq. (34) then for $P_i(\Phi, \varepsilon)$ one obtains the reaction formula

$$P_n = P_{n-1} \sin \Phi + \frac{1}{\varepsilon} \frac{d P_{n-1}}{d \Phi}$$

and with consideration of the first four terms, the approximate solution

$$P(\Phi, \gamma, \varepsilon) = C_2 \left\{ \frac{1}{1 - \gamma \sin \Phi} + \frac{\gamma^2}{\varepsilon \cos \Phi} + \gamma^3 \left(\frac{1}{\varepsilon \cos^2 \Phi \sin \Phi} - \frac{1}{\varepsilon^2 \sin \Phi} \right) \right\} \quad (35)$$

whereby C_2 is determined from the standardization condition

$$\int_{-\pi}^{+\pi} P(\Phi, \gamma, \varepsilon) d\Phi = 1$$

to be

$$C_2 = \frac{\sqrt{1 - \gamma^2}}{2\pi}$$

With the density distribution for the phase error obtained in this manner, according to Eq. (24) the noise-dependent continuous component can now be determined. For this a check is first made to see whether this procedure, which uses the ensemble mean value, yields usable results in the noise-free case. If in the initial form of Eq. (31) one sets the noise term equal to zero, then after integrating it and determining the constants one finds the distribution

$$P(\Phi) = \frac{\Delta \omega_0 \sqrt{1 - \gamma^2}}{(\Delta \omega_0 - a K \sin \Phi) \cdot 2\pi} \quad (36)$$

The standardized continuous component < sin Φ > is calculated with Eq. (36) to be

(37) $$<\sin\Phi>_{RF}=\int_{-\pi}^{+\pi}\sin\Phi\, P(\Phi)\,d\Phi=\frac{1}{\gamma}\left(1-\sqrt{1-\gamma^2}\right)$$

This d-c voltage curve mirrors the behavior of $<\sin\Phi>_{RF}$ as a function of $\Delta\omega_0$ better than the expression found from Eq. (12) because it does not show the discontinuity for γ = 1 (see Fig. 3). By the way, both function curves are identical at least for $\Delta\omega_0>2\Delta\omega_L$) .

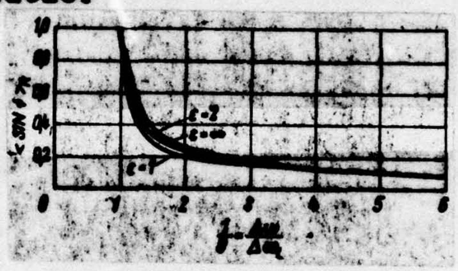
Because of this unique result in the case of the noise-free input signal the noise-dependent continuous component is now calculated. If one substitutes the distribution given by Eq. (35) in Eq. (24) with the limits +-π, then following determination of the integration constants one finds

(38) $$<\sin\Phi>_R=<\sin\Phi>_{RF}-\frac{\sqrt{1-\gamma^2}\cdot\gamma^2}{2\varepsilon^2}$$

$<\sin\Phi>_{RF}$ represents the continuous component given by Eq. (37) in the noise-free case. This result shows that in the case of a

worsening signal-to-noise ratio due to the second summand there results a certain reduction of the continuous component and therewith a lengthening of the acquisition times. If one considers, however, that for ε during observation of the output limit of the automatic control in no case do values of $\varepsilon < 1$ come into question and that the largest value of γ is given with $\gamma = 1/2$ on the basis of the approximation in §3, then the conclusion must be drawn that for all cases of practical interest $\langle \sin \Phi \rangle_R$ is essentially independent from the signal-to-noise ratio (see Fig. 6).

Fig. 6. Dependence of the standardized continuous component on the signal-to-noise ratio.



Since the dependence calculated here is far from sufficient for explaining the lengthening of acquisition times found in the experiment in the case of decreasing ε, it must be assumed that the noise acts primarily on the second partial process which was explained in §3. The calculation of this influence will be the task of the following section.

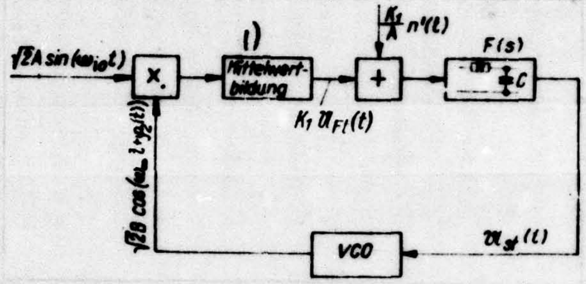
5.2. The Noise-Dependent Acquisition Equation

The fundamental concept of the quasi-stationary approximation in §3 consisted of the division of the operating principle of the second order automatic control system into two activities: continuous component formation and the continuous shifting of the VCO-mean frequency in the direction of the signal frequency caused by it. These partial processes should also determine the acquisition process in the case of a noisy input signal, whereby they are then affected

by the noise. If one further holds on to the assumption that the operating mode of the multiplier is independent of the signal-to-noise ratio then on the basis of Eq. (38) the noise can make itself apparent only on the second partial process.

For mathematical description of the present noise effect we shall use the equivalent circuit diagram (Fig. 7).

Fig. 7. Fundamental circuit diagram of the modified second-order phase automatic control system. KEY: 1) mean value formation.



One imagines the core of the noise as connected to the circuit only after completed formation of the continuous component so that the instantaneous frequency deviation thus becomes a random variable, as was the case with $\bar{\Phi}(t)$ in Eq. (23). The new and now, noise-dependent, acquisition equation thus reads

(39)
$$\tau_1 \frac{d\,\Delta\omega(t)}{dt} + \Delta\omega(t) - \Delta\omega_0 = -K\left(\frac{a \cdot K}{2\,\Delta\omega(t)} - \frac{n'(t)}{A}\right)$$

This process also represents a continuous Markov process to which once again, making certain assumptions, a Fokker-Planck equation can be assigned and whose solution provides the distribution density function $P(\Delta\omega, t)$ for the possible frequency deviations.

Since we are once again dealing with a differential equation of the first order and since the noise process $n'(t)$ is to have the same

properties as before, in

(40) $$\frac{\partial P(\Delta\omega, t)}{\partial t} = \sum_{1}^{\infty} n \frac{(-1)^n}{n!} \frac{\partial^n}{\partial \Delta\omega^n} [A_n(\Delta\omega) P(\Delta\omega, t)]$$

once again all $A_n(\Delta\omega)$ vanish for n > 2. Its significance is the same as in Eq. (26) and its calculation yields

(41) $$A_1(\Delta\omega) = \frac{1}{\tau_1}\left\{\Delta\omega_0 - \left(\Delta\omega + K \cdot \frac{aK}{2\Delta\omega}\right)\right\} = -\lambda(\Delta\omega)$$

and

(42) $$A_2(\Delta\omega) = \frac{K^2 N_0}{2A^2\tau_1^2} = 2\varkappa$$

With these two expressions the partial differential equation (40) converts into the special form

(43) $$\frac{\partial P(\Delta\omega, t)}{\partial t} = \frac{\partial}{\partial \Delta\omega}(\lambda(\Delta\omega) \cdot P(\Delta\omega, t)) + \varkappa \frac{\partial^2 P(\Delta\omega, t)}{\partial \Delta\omega^2}$$

Before the Fokker-Planck equation can be solved some remarks must be made on the beginning-, boundary- and standardizing conditions.

The beginning conditions are initially of interest because since

we are talking about the distribution of frequency error we need the time-dependent solution of Eq. (43). For the sake of simplicity in calculation we shall not begin the process from a delta distribution but rather from a narrow Gaussian distribution, whereby the static values are taken from the experiment. Its general form reads

$$P(\Delta\omega, 0) = \frac{1}{\sqrt{2\pi}\,\tau_0} \exp - \left\{ \frac{(\Delta\omega - \overline{\Delta\omega_0})^2}{2\tau_0^2} \right\} \tag{44}$$

with $\overline{\Delta\omega_0}$ as the mean initial deviation and τ_0 as the dispersion of the $\Delta\omega$-values of each 100 measurements conducted at the point in time $t = 0$.

Somewhat more difficult is the formulation of the boundary conditions. On the basis of the definition of the Ranlauf duration t_A the acquisition is considered as ended if the the frequency error limit $\Delta\omega = \Delta\omega_L$ is reached. That means that all densities for the states $\Delta\omega < \Delta\omega_L$ must subsequently be set equal to zero. Thus the first boundary condition reads

$$P(\Delta\omega \leq \Delta\omega_L; t) = 0 \text{ für alle } t \tag{45}$$

As long as one is at the beginning of the acquisition procedure the demand of Eq. (45) is not critical since the distributions are still

relatively narrow and their expected values with respect to $\Delta\omega_L$ are quite large. With increasing duration, however, P ($\Delta\omega$, t not only spreads, but $\Delta\omega$ also drifts in the direction of $\Delta\omega_L$, whereby the discontinuity conditioned by the cutting off of the values of the distribution density at $\Delta\omega_L$ beginning from a certain point in time can lead to considerable errors in the total distribution. So that these errors do not occur a criterion must be determined, with which one decides from case to case how long a computational observation of the Ranlauf process can continue.

The limitation of the permissible $\Delta\omega$-interval to greater values than $\overline{\Delta\omega}$ can take place quite arbitrarily. As a maximum $\Delta\omega_{max}=\Delta\omega_p$ could be chosen. For saving calculation steps $\Delta\omega_{max}$ is made variable here and depending on the initial deviation and the initial dispersion is kept as small as possible.

A unique formulation of the standardizing condition is not possible on the basis of these statements. However, in this case that is not a problem, since Eq. (43) must be solved numerically so that one can do without the standardizing condition.

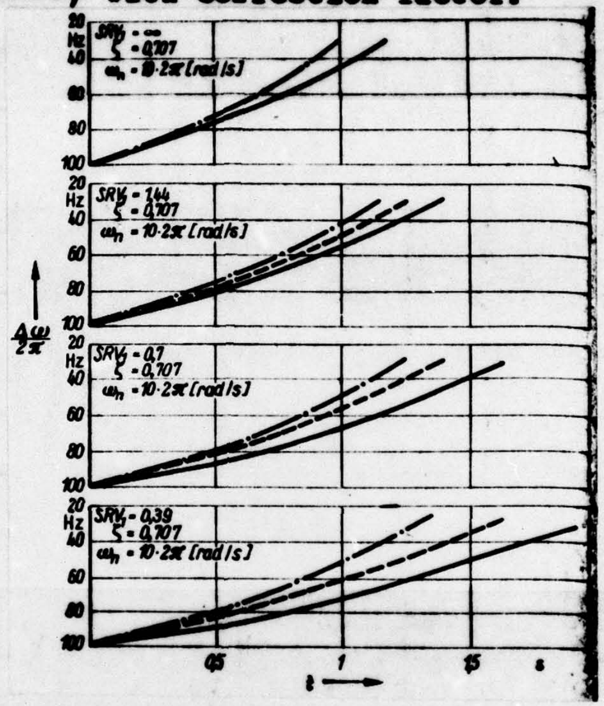
Using the difference method the previous boundary value problem converts into an algebraic difference equation with respect to $P_{m,n}$ of the form

$$P_{m,n+1} = \frac{l \cdot x}{h^2} P_{m-1,n} + \Big\{1 + \frac{l}{\tau_1} - \frac{g \cdot l}{\tau_1 \Delta\omega^2} - \frac{2xl}{h^2} - \frac{l}{h\tau_1}\left(\Delta\omega + \frac{g}{\Delta\omega} - \Delta\omega_0\right) P_{m,n} + \frac{l \cdot x}{h^2} + \frac{l}{h \cdot \tau_1}\left(\Delta\omega + \frac{g}{\Delta\omega} - \Delta\omega_0\right)\Big\} P_{m+1,n} \tag{46}$$

Here l and h are the step widths in the t-, respectively $\Delta\omega$-direction; g is the abbreviation $g = \alpha \cdot K^2/2$.

Corresponding to the experiment several acquisitions were calculated in this manner for several signal-to-noise ratios, initial deviations and ζ- and ω_n- values. Fig. 8 shows typical results. Some frequency time distributions are shown for acquisitions with an initial deviation of $\Delta\omega_0 = 98.4 \cdot 2$ rad/s with four different signal-to-noise ratios and the circuit parameters $\zeta = 0.707$ and $\omega_n = 10 \cdot 2\pi$ rad/s. The experimental results were determined from the recording of 100 error voltage distributions $U_F(t)$ in each case. In accordance with §3 a frequency time distribution was assigned to each individual fluctuation on the multiplier output; then a mean distribution was determined from all of them (in each case, the solid lines).

Fig. 8. Experimental (———) and theoretical frequency time distributions of a 98.4-Hz acquisition (-•-•-•-•) without correction factor and (-----) with correction factor.



The curves drawn with dash-dot lines represent the theoretical results. The expected values were extracted from the calculated distribution functions and plotted as functions of time. It turns out that its drift reflects the experimentally found behavior relatively well down to input signal-to-noise ratios of $SRV_1 \approx 2$. In addition we recognize that in the range $\infty \geq SRV_1 > 2$ the acquisition delay caused by noise is small compared to the noise-free case. It does not exceed

15 $^{0}/_{0}$ in either the experiment or in theory.

If, on the other hand, SRV_1 drops to values of 1 and below it turns out that the theoretically found effect of the noise on the acquisition behavior is not sufficient to explain the experimental results. The deviation between the frequency time responses rapidly becomes larger as SRV_1 becomes smaller.

These results which initially are only valid for the represented 98.4-Hz acquisitions with the given parameters can be generalized even more. Both experimentally and theoretically determined transient responses for other initial deviations from the range $2\Delta\omega_L < \Delta\omega_0 < \Delta\omega_p$ demonstrated roughly the same behavior with respect to the noise effect, whereby the deviation for smaller $\Delta\omega_0$ is somewhat less than for large $\Delta\omega_0$.

It thus remains to be explained what causes the in part considerable acquisition time delays in the experiment during approach of the signal-to-noise ratio to the value 1. If one wants to hold on to an ideal, i.e., noise-independent, operating mode of the individual components of the automatic control system then the described theory can no longer work. Due to the quasi-stationary approximation method the acquisition process was split into two partial processes and in §5.1 and §5.2 it was assumed that the noise

only affected one of the partial processes in each case. Other possibilities for effect do not exist under these conditions so that the larger delays with small SRV_1 cannot initially be explained.

5.3 The Corrected Noise-Dependent Acquisition Equation

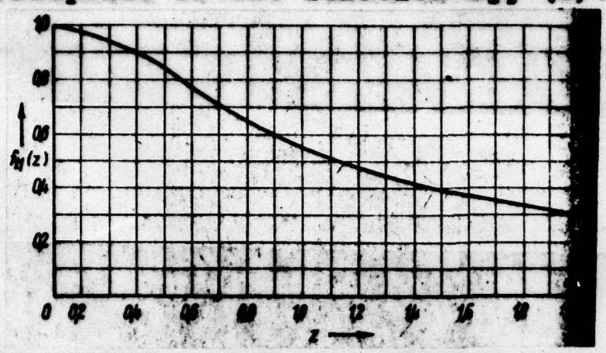
In the experiment a ring modulator served as a multiplier of the phase automatic control system. From [6], however, it was shown that the operating mode of a ring modulator becomes a function of noise when the SRV_1 affecting it approach values of 1 and less. Given a bending characteristic for the diodes (amount rectification) and with the assumption that the employed noise process is Gaussian the efficiency of such a circuit was calculated as a function of the signal-to-noise ratio and the alternating voltage amplitudes A and B. Since the measuring installation employed a semiconductor ring modulator with symmetry resistances and since the current through the diodes in general is determined not by its internal resistance but by that of the source [6], the results can be carried over to the acquisition observations. Accordingly, at the multiplication output as an error voltage $U_F(t)$ there results

$$U_F(t) = K_1 \cdot f_{11}(s) \cdot \sin \Phi(t) \qquad (47)$$

where the higher order terms in $\sin \Phi(t)$ were disregarded. The

connection between f_{11} and z is given in the aforementioned work. Here we shall only go into the results of the calculations which were performed there. For the value interval $0 \leq z \leq 2$ the function response results which is shown in Fig. 9.

Fig. 9. Response of the function f_{11} (z).



According to

(48) $$z = \frac{\sigma}{\sqrt{2(A^2 + B^2)}}$$

the value z with σ as effective value of the noise voltage is not only a function of the signal-to-noise ratio but also depends on the size ratio of the signal voltages A and B arriving at the multiplier. With a given VCO-voltage, the smaller the signal-to-noise ratio, the smaller will be value $f_{11}(z)$ according to Fig. 9. For the continuous component $\bar{U}_F$ of the error voltage according to

(49) $$\bar{U}_F = K_1 \cdot f_{11}(z) \cdot \overline{\sin \Phi}$$

this means that due to the noise-dependent operating mode of the

multiplier there is a considerable reduction of the acquisition speed in comparison to the results of the previous section (dot-dash curves in Fig. 8).

For quantitative formulation of this result the correction factor $f_{11}(z)$ was built into the partial differential equation (43) whereby instead of K_1 the value $K_1 \cdot f_{11}(z)f$ which is dependent on SRV_1 appears as the amplitude factor of the d-c component. Corresponding to the experimental conditions and voltage relationships the cases from 5.2 were recalculated. The thus obtained frequency-time responses are shown by the middle curves in Fig. 8 (dotted lines). It is evident that the agreement between the theoretical and experimental acquisition times is considerably better than was the case when the values were calculated without the correction factor. The maximum deviations which occur are now smaller than 20 $^0/_0$ in all of the examined cases. If one notes that even in the noise-free case there is a certain discrepancy between the two frequency-time responses, it turns out that the theoretically formulated effect of the SRV_1 on the acquisition process describes the experimentally found behavior even with deviations of a maximum of only about 15 $^0/_0$. Therewith the cause of the behavior of the experimental lag process with small SRV_1 seems to become explainable. The practician who wishes to design an automatic control system based on acquisition times thus acquires the possibility of evaluating the expected

acquisition times not only for signal-to-noise ratios down to values of of $SRV_1 \approx 2$ but also for much smaller values.

6. Conclusions

The results of the preceding observations show that the quasi-stationary approximation method used here seems suitable for describing distant acquisition (acquisition from outside of the pull-in range) of a second-order phase automatic control system both in the case of noisy- and in the case of noise-free input signals. In the noise-free case we find the same results as are familiar in the literature. As long as the input signal-to-noise ratio does not become smaller than 10 the noise seems to have little effect on the synchronization process. This is also confirmed experimentally. A delay for the acquisition times by about 20 % compared to the noise-free case resulted first for an SRV_1 value of about 2. Up to this range of values the acquisition equation with the correcting term according to Eq. (43) for practical purposes seems to describe the experimental Ranlauf process well enough.

Unfortunately the relatively small effect of the noise on acquisition at even smaller signal-to-noise ratios could not be confirmed experimentally since in this range a problem developed with efficiency deterioration of the ring modulator which did not meet the

prerequisites for correction-free calculation. The consideration of this effect in §5.3 could explain the large delays which appeared in the experiment only within a 20 $^{0}/_{0}$ error limit.

Nevertheless the results of calculations without a correcting term seem plausible even if they couldn't be confirmed with the multiplier parameters employed here. If on the multiplier (ring modulator), however, one selects a V/A ratio so large that in spite of the small SRV_1 values the value $f_{11}(x) \approx 1$, then its experimental confirmation in this range of signal-to-noise ratios should be possible.

The results obtained in §5.2 and §5.3 with respect to the effect of the noise on the acquisition process are not limited to just the transient effect in the middle of the pull-in range. The treatment of acquisitions from other initial deviations from the range $\Delta\omega_L < \Delta\omega_0 < \Delta\omega_p$ showed roughly the same noise effect. An exception are the results which were obtained for the initial deviatons in the vicinity of the pull-in frequency. On the basis of the approximation made in §4.3 (D >>1) somewhat greater differences arise here between the theoretical and experimental results.

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LITERATURE

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